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THE INFLUENCE OF A HIGH PRESSURE GRADIENT ON
UNSTEADY VELOCITY PERTURBATIONS IN THE CASE OF
A TURBULENT SUPERSONIC FLOW

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16. Abstract
The amplification or reduction of unsteady velocity perturbations under the influence of strong flow acceleration or deceleration is studied. Supersonic flows with large velocity and pressure gradients are examined and the conditions in which the velocity fluctuations depend on the action of the average gradients of pressure and velocity, rather than turbulence, are described. Experimental results obtained on a cylinder positioned parallel to the flow are analyzed statistically and interpreted as a "return to laminar" process. It is shown that this "return to laminar" implies negative values in the turbulence production terms for kinetic energy. A simple geometrical representation of the Reynolds stress production is given. It is also shown that in the experimental case the influence of viscous energy dissipation cannot be neglected.

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J. P. Dussauge and J. F. Debieve

I. Introduction

The amplification or reduction of stationary velocity perturbations under the effect of strong accelerations or decelerations of a permanent average flow is studied. The cases examined relate to supersonic flows in which there are high velocity and pressure gradients. In these conditions, the evolution of the turbulent velocity fluctuations processed by the statistical method, presents special aspects, which we attempt to analyze here. First of all we will define the conditions in which the evolutions considered are not entirely specific of the turbulence, then the experimental results obtained in a flow undergoing expansion are described. These results, which have already been published are analyzed under a new aspect: they are compared with a relaminarization criterion; the latter is interpreted in terms of "production of turbulence", of which a few analytical elements are given. Finally we make an estimate showing that in the studied expansion, the effect of the non linear and viscous terms involved in the kinetic energy equation of turbulence does not appear predominant.

II. Analysis of the Problem

We consider a permanent average flow, on which unstationary fluctuations of temperature, pressure, volumic mass and speed are superimposed. The level of these fluctuations is low as compared with that of the average quantities; furthermore it may vary under the effect of gradients of average velocity and pressure. In this

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*Numbers in the margin indicate pagination in the foreign text.

study, we are particularly interested in the velocity fluctuations, which will be processed statistically, and in their evolution as a function of the variations of the average field. In certain conditions, this evolution may show aspects which are not totally specific of turbulence. The object of this paragraph is to specify these conditions.

Let us consider the equation of kinetic energy of turbulence, established with the use of average mass weighted quantities.

$$\bar{\rho}\frac{\partial}{\partial t}\frac{1}{2}\frac{\overline{\rho u_i' u_i'}}{\bar{\rho}} + \bar{\rho}\tilde{u}_j \frac{\partial}{\partial x_j}\frac{1}{2}\frac{\overline{\rho u_i' u_i'}}{\bar{\rho}} = \underbrace{\qquad\qquad\qquad\qquad\qquad}_{I} \qquad\qquad (1)$$

$$-\overline{\rho u_i' u_j'}\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\overline{p' u_i'}}{\bar{\rho}}\frac{\partial \bar{p}}{\partial x_i} - \overline{u_i'\frac{\partial p'}{\partial x_i}} + D - \overline{\varphi'}$$

$$\qquad\quad II \qquad\qquad\quad III \qquad\qquad IV \quad\; V \;\; VI$$

x_i represents the space variables, u_i the velocity components, ρ the volumic mass, p the pressure. The symbol $\overline{}$ represents an average, $\sim$ a mass weighted average and $'$ a fluctuation. The mass weighted average of a quantity w is defined by

$$\bar{\rho}\tilde{w} = \overline{\rho w} \quad \text{which implies} \quad \overline{\rho w'} = 0 .$$

The different terms of equation (1) may be interpreted as follows (J. Gaviglio, J. P. Dussauge, J. F. Debieve, A. Favre, 1977): the term I represents the variation of $\overline{\rho u_i' u_i'}/\bar{\rho}$ along an average current line; II is a term of "kinetic" production; III represents an "enthalpic" production; the term IV, for a solenoidal velocity fluctuation field ($\partial u_i'/\partial x_i = 0$) is usually grouped with the diffusion term V; the term VI designates energy dissipation under the effect of viscosity.

Seeing that in the accelerated or decelerated areas in

supersonic flow, $\overline{p}$ may vary greatly, we will consider the evolution of $\overline{\rho u_i' u_i'}/\overline{\rho}$, which is equal to $\overline{u_i'' u_i''}$ if the term $\overline{\rho' u_i'' u_i''}/\overline{\rho}$ of the third order with respect to the fluctuations, is negligible.

In the turbulent flows which undergo little change, diffusion and dissipation play an important role. For example, in a boundary layer in equilibrium, the terms of production, diffusion and dissipation are of the same order of magnitude. The latter is equal to $\overline{\rho} q'^3/\Lambda$, if it is assumed that the dissipation has the same form in compressible and incompressible flows, $q' = \sqrt{\frac{1}{3} \overline{u_i' u_i'}}$ and Λ is a space scale characteristic of energy bearing turbulent structures, for example a space scale derived from correlation (Tennekes and J. L. Lumley, 1972).

In a distortion region, that is, wherever the average flow varies to a considerable extent, the order of magnitude of the production terms may be changed. It happens that for moderate Mach numbers ($M \leqslant 4$), the terms of kinetic and enthalpic production have comparable values (J. P. Dussauge, J. Gaviglio, A. Favre, 1978). Their order of magnitude may be estimated by $\overline{\rho} q'^2 \Delta U/L_d$, in which ΔU and L_d designate a characteristic scale of the evolution of the average velocity and a space scale relating to distortion respectively.

If the terms of diffusion and dissipation do not vary much in the distortion, they can be disregarded, on the condition that:

$$\overline{\rho} q'^2 \frac{\Delta U}{L_d} \gg \overline{\rho} \frac{q'^3}{\Lambda}$$

which is equivalent to writing:

$$\frac{q'}{\Delta U} \frac{L_d}{\Lambda} \ll 1 \qquad (2)$$

We may estimate the condition for which the dissipation rate will not vary much: it is sufficient that the time during which a

fluid particle undergoes distortion should be less than its "characteristic time scale" (or "lifetime") q'/Λ. Therefore

$$\frac{L_d}{U} \ll \frac{\Lambda}{q'}$$

or

$$\frac{q'}{U} \frac{L_d}{\Lambda} \ll 1 \qquad (3)$$

(see for example, Batchelor, 1967)

U is an estimate of the local average velocity within the distortion. As regards the diffusion term, it must be considered separately in each special case (for example, see Paragraph V. 1.).

If the inequalities (2) and (3) apply and if the diffusion rate varies little, this would mean that the flow is such that the influence of the effects of the diffusion and dissipation terms, specific of turbulence, is negligible. In this case, we find a type of flow in which the evolution of the level of unstationary fluctuations is relatively independent of the classical properties of turbulence.

III. Studied Flow

III. 1. Description

The flow in which the variations of the statistical level of the fluctuations were observed is the close wake developing downstream of a truncated cylinder placed parallel to the flow (Fig. 1). This configuration was already described in previous publications (J. Gaviglio, J. P. Dussauge, J. F. Debieve, A. Favre, 1977, J. P. Dussauge, J. Gaviglio, A. Favre, 1978). Its main characteristics are recalled below.

The pressure generating the potential flow is 0.375 atmosphere.
The Mach number is 2.3 in the external flow, upstream of the base

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of the cylinder. A fully developed turbulent boundary layer
develops on the cylinder. The boundary layer, near the end of
the cylinder, is subject to expansion. There is detachment,
with the constitution of a recirculation area surrounded by a
mixing zone. Recompression takes place during which there is
confluence of the mixing area. The wake is formed downstream.

The region which we will be discussing chiefly is the
boundary layer undergoing expansion.

III. 2. Measurements Carried Out

III. 2. 1. The results described are of two types:
measurements of average quantities by pressure probes and measure-
ments of standard deviations and correlation coefficients relating
to velocity and temperature fluctuations. In the following, the
equations are written in a system of coordinates bound with the
average current lines. The direction s is taken along their
tangent, y along the normal, ψ along the binormal. The only
component of the average velocity is $\tilde{u}$ and the velocity fluctuations
in the three previous directions are noted u', v', w'; θ
designates the temperature; r the distance in a given point and
the axis of the cylinder of diameter d ; η refers to the average
current line of zero rate of flow.

III. 2. 2. Initial Boundary Layer

The characteristics of the boundary layer upstream of
the expansion are as follows: its thickness $\delta(o)$ is 4 mm; the
Reynolds number referred to the thickness of the quantity of
motion is 10^3. The friction coefficient, derived from the average
measurements is $C_f = \cdots \rho_o U_{\infty} \quad \approx 2,9. \ 10^{-3}$. τ_o is the friction
on the wall; the index ∞ relates to the external flow.

Standard deviation measurements of velocity and
temperature fluctuations were carried out with a hot wire anemometer,
of the "constant intensity" type. Because of difficult measurement
conditions, the measurements were corrected by an adapted method

(J. Gaviglio, J. P. Dussauge, 1977) and compared with the known
results relating to boundary layers in equilibrium (Fig. 2).
The measurements carried out do indeed correspond to conditions
of boundary layer in equilibrium.

III. 2. 3. Expansion Area

In the expansion area, measurements of pressure and
average velocity were carried out, making it possible to estimate
the gradients of these quantities. An example is given by
Figure 3, which presents profiles of average velocity. In the
expansion, the results of the measurements were compared with
the calculated profile assuming that along an average current line,
the evolution is isentropic and the friction forces are small as
compared with the pressure forces. In these conditions the first
equation of the quantity of motion is written, in a system of
coordinates bound with the average current lines:

$$\overline{\rho}\,\tilde{u}\,\frac{\partial \tilde{u}}{\partial s} = -\frac{\partial \overline{p}}{\partial s}$$

It is apparent that there is satisfactory consistency
between measurement and calculation. The shape of the profiles
obtained shows the effect of the expansion on the average velocity
gradients: in particular there is creation of a positive gradient
$\partial \tilde{u}/\partial s$ and considerable reduction of $\partial \tilde{u}/\partial y$.

As regards the pressure gradients, we find a negative
longitudinal gradient $\partial \overline{p}/\partial s$ and a positive $\partial \overline{p}/\partial y$ transversal
gradient.

Measurements of standard deviations of velocity
fluctuations were carried out in expansion (Fig. 4). The results
show that in this area there is a considerable drop in the level
of the longitudinal velocity fluctuation. Analytical elements

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which make it possible to relate this evolution with that of the average gradients are proposed in paragraphs IV. 1. and V.

III. 2. 4.   Results Common to Any Flow

The measurements of temperature fluctuation and of the correlation coefficient between the longitudinal component of the velocity and temperature show that in any flow, including accelerations and decelerations:

1.   the formula derived from "Strong Reynolds Analogy" applies (Fig. 5):

$$\frac{\sqrt{\overline{\theta'^2}}}{\overline{\theta}} = (\gamma - 1) M^2 \frac{\sqrt{\overline{u'^2}}}{\overline{U}}$$

M is the Mach number and γ the ratio of the specific heats of the gas.

2.   The correlation coefficient between velocity and temperature is practically constant and its value is close to -0.8 (Fig. 6) which is close to the values measured in heated subsonic flow. These results will be used in paragraph IV. 1. 2.

IV.   Interpretation of the Results of Measurements

IV. 1.   Reminders

IV. 1. 1.   Criterion of Narasimha and Visvanath

Since the studies of Sternberg (1954), we know that boundary layers exposed to considerable accelerations, especially in centered expansions, in supersonic flow, have a tendency to "relaminarization." This relaminarization is expressed in a decrease of the intensities of turbulence, and a variation of the value of the parameters like the parietal thermal factor. Narasimha and Visvanath (1975) reviewed the experiments in which relaminarization could occur, in view of establishing simple criteria making it possible to foresee the cases of "relaminarization."

These authors have reasoned as follows: they assume that in the expansion, the friction forces are small as compared with the pressure forces. Furthermore, they take into account the fact that in the expansions observed, the length over which the latter are spread are of the order of the thickness δ of the boundary layer. The longitudinal gradient may therefore be estimated as $-\Delta p / \delta$, in which Δp is the absolute value of the pressure deviation on either side of the expansion. If the pressure forces are great enough, relaminarization takes place. The ratio of these two forces may be estimated as follows:

$$\frac{|\partial \bar{p}/\partial s|}{\partial \tau/\partial y} \sim \frac{\Delta p/\delta}{\tau_0/\delta} \sim \frac{\Delta p}{\tau_0}$$

τ_0 is the wall friction of the initial layer.

Reviewing the measurements conducted by different authors, Narasimha and Visvanath find that for $\Delta p/\tau_0 > 70$, there is return to the laminar state, and that for $\Delta p/\tau_0 < 60$, this return is not mentioned.

IV. 1. 2. Production of Turbulence

The decrease of velocity fluctuations in the expansion of the near wake flow has been attributed in previous publications (J. Gaviglio, J. P. Dussauge, J. F. Debieve, A. Favre, 1977; J. P. Dussauge, J. Gaviglio, A. Favre, 1978) to negative values of terms of "production of turbulence", present in the equations of kinetic energy of the turbulence and in the equations of the Reynolds tensions. Let us recall briefly the main results.

The terms of production are given as:

$$P = -\frac{\overline{\rho u'_i u'_j}}{\bar{\rho}} \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\overline{\rho' u'_i}}{\bar{\rho}} \frac{\partial \bar{p}}{\partial x_i}$$

P will be designated as the simple term of "production", to differentiate it from kinetic and enthalpic productions.

P was estimated with the following hypotheses:

  a. The field of velocity fluctuations is solenoidal $(\partial u_i'/\partial x_i = 0)$.

  b. The temperature fluctuations are practically isobaric:

$$\frac{\rho'}{\rho} \simeq -\frac{\theta'}{\Theta} \qquad \text{(Laufer 1969)}$$

  c. The values of the correlation coefficients between the standard deviations of the different velocity components, between the fluctuations of velocity and temperature are not very different from what they are in the boundary layer in equilibrium.

  d. The values of the ratios between the standard deviations of the fluctuations of the different velocity components are close to those found in the equilibrium boundary layers.

  e. The standard deviations of the longitudinal fluctuations of velocity and the temperature fluctuations satisfy the relation given by the "Strong Reynolds Analogy" mentioned in paragraph III. 2.

Furthermore the terms of production of turbulence relating to each of the variances of velocity fluctuation $\overline{u'^2}$, $\overline{v'^2}$, $\overline{w'^2}$, have been given in a coordinate system bound with average current lines. Taking into consideration the average geometry of the flow and the direction of mass transfers by turbulence, it may thus be shown that the absolute value of the production of $\overline{w'^2}$ is negligible as compared with those of $\overline{u'^2}$, and $\overline{v'^2}$; furthermore, the production of $\overline{v'^2}$ is always negative. Therefore in this case if the production of $\overline{u'^2}$ is negative, the same will hold good for the production of $\overline{u'^2}$, $\overline{v'^2}$, $\overline{w'^2}$. (Dussauge, Gaviglio, Favre, 1978.)

The production of turbulence P_3 relating to $\overline{u'^2}$ is written:

$$P_3 = - \frac{\overline{\rho u' v'}}{\overline{r}} \frac{\partial \tilde{u}}{\partial y} - \frac{\overline{\rho \tilde{u}^2}}{\overline{\rho}} \frac{\partial \tilde{u}}{\partial s} + \frac{\overline{\rho' u'}}{\overline{\rho}} \frac{\partial \tilde{\rho}}{\partial s}$$

The only positive term is $- \frac{\overline{\rho u' v'}}{\overline{r}} \frac{\partial \tilde{u}}{\partial y}$. But Figure 2 shows that in the expansion, $\partial \tilde{u}/\partial y$ decreases considerably. With the hypotheses put forward previously, it is found that the terms $\overline{\rho u' v'} \partial \tilde{u}/\partial y$ and $\overline{\rho \tilde{u}^2} \partial \tilde{u}/\partial s$ are of opposite signs and of the same order of magnitude /10 in absolute value. The negative term $\overline{\rho' u'} \partial \tilde{\rho}/\partial s$ is then highest in absolute value, it gives then its sign to P_3. Furthermore we found consistency in the order of magnitude between the estimate of P_3 and the measured rate of decrease of $\overline{u'^2}$. Thus the evolution observed is explained to a considerable extent by the negative values of the term of production of turbulence.

IV. 2. Interpretation of the Criterion of Relaminarization of Narasimha and Visvanath as a Function of the Production of Turbulence

The guiding principle of Narasimha and Visvanath in the establishment of their criterion is that the pressure forces are large as compared with friction forces caused by turbulence, in an expansion. This is an argument presented in terms of quantity of motion, which provides little information on the evolution of the kinetic energy of agitation. It is shown here that the criterion of Narasimha and Visvanath may be associated with negative values of the production of $\overline{u'^2}$, therefore production of turbulence. Let us consider again the expression of P_3 relating to $\frac{\overline{\rho u'^2}}{\overline{\rho}} \simeq \overline{u'^2}$.

$$P_3 = - \frac{\overline{\rho u' v'}}{\overline{r}} \frac{\partial \tilde{u}}{\partial y} - \frac{\overline{\rho u'^2}}{\overline{\rho}} \frac{\partial \tilde{u}}{\partial s} + \frac{\overline{\rho' u'}}{\overline{\rho}^2} \frac{\partial \tilde{\rho}}{\partial s}$$

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If the friction forces are neligible as compared with the pressure forces, we have (Paragraph III. 2. 2.):

$$\rho u \frac{\partial \tilde{u}}{\partial s} = - \frac{\partial \bar{p}}{\partial s}$$

Using the relations:

$$\frac{\sqrt{\overline{\theta'^2}}}{\hat{\theta}} = \frac{\sqrt{\overline{p'^2}}}{\bar{p}} = (\gamma-1) M^2 \frac{\sqrt{\overline{u'^2}}}{\tilde{u}}$$

and

$$r_{\theta u} = - r_{\rho u} = - \frac{\overline{\rho' u'}}{\sqrt{\overline{\rho'^2}} \sqrt{\overline{u'^2}}}$$

we obtain

$$P_s = - \frac{\overline{\rho u' v'}}{\bar{p}} \frac{\partial \tilde{u}}{\partial y} + \frac{\overline{u'^2}}{\bar{p}\tilde{u}} \frac{\partial \bar{p}}{\partial s} \left[1 - (\gamma-1)M^2 r_{\theta u} \right]$$

The second term is negative. P_s is also negative if:

$$\frac{\overline{u'^2}}{\tilde{u}} \left| \frac{\partial \bar{p}}{\partial s} \right| \left[1 - (\gamma-1) M^2 r_{\theta u} \right] > - \overline{\rho u' v'} \frac{\partial \tilde{u}}{\partial y}$$

Because of the geometry of the expansion area, we have $|\partial \bar{p}/\partial s| \sim$ $\Delta p / \delta$, in which Δp is the difference of pressure on either side of the expansion.

Likewise

$$- \overline{\rho u' v'} \frac{\partial \tilde{u}}{\partial y} \sim \tau_0 \frac{u_\infty}{\delta}$$

It was noted that τ_0 is probably an upper bound of $-\overline{\rho u' v'}$, and u_∞/δ is also an upper bound of $\partial \tilde{u}/\partial y$ (Fig. 3), outside the wall law:

We find:

$$\frac{\Delta p}{\tau_0} > \frac{u_\infty^2}{\overline{u'^2}} \frac{1}{1 - (\gamma-1)M^2 r_{\theta u}}$$

For $M = 2$, $r_{\rho u} = -0.8$ and $\dfrac{\sqrt{\overline{u'^2}}}{u_\infty} \simeq 6 \cdot 10^{-2}$

$$\frac{\Delta \tau_\nu}{\tau_0} \,\, > 1 \simeq 0$$

If on the other hand, we calculate $\Delta \tau_\nu / \tau_0$, in the case of our experiment, we find the value 46; we revealed here a negative production of turbulence. The condition $\Delta \tau_\nu / \tau_0 > 120$ appears therefore to be superfluous. This is no doubt due to the considerable excess taken when it is assumed that $\partial \overline{u} / \partial y$ is of the order u_∞ / δ. Actually Fig. 3 shows that in the accelerated region, we would be having rather $\partial \overline{u} / \partial y \sim 0.1 \, u_\infty / \delta$. Furthermore the value 70 given by Narasimha and Visvanath is really of the same order of magnitude as the two above-indicated estimates, which correspond to negative values of the production term.

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IV. 3.   Remarks on the Term of Kinetic Production

In the preceding, we discussed mainly the production of the kinetic energy of turbulence. But the analysis of the behavior of this term led to considering the production of the normal Reynolds tension $\overline{\rho u'^2}$. A few characteristics are given here of the effect of the kinetic production of Reynolds tensions. It is found that it is difficult to evaluate the latter. But it is possible to obtain a simple geometrical representation of it taking into account the geometry of the average flow.

To do this, let us consider the quantity $Q = \overline{\rho (\underline{v'} \cdot \underline{N})^2} / \overline{\rho}$ representative of the agitation in the direction of a vector $\underline{N}$. The evolution with the average movement according to the law (Debieve, 1978) is imposed on the vector $\underline{N}$:

$$\frac{D}{Dt} N_i = N_j \frac{\partial}{\partial x_i} \overline{u}_j \quad \text{with} \quad \frac{D}{Dt} = \frac{\partial}{\partial t} + \overline{u}_j \frac{\partial}{\partial x_j}$$

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In the case of a pure deformation, this corresponds to the ordinary concept of the transport of a vector by the average movement: $\underline{N}$ would be carried away by the fluid with contraction or dilation according to the deformation of the average movement. The evolution of Q is given by:

$$\bar{\rho}\frac{D}{Dt}\frac{\overline{\rho(\underline{v}'\cdot\underline{u})^2}}{\bar{\rho}} = N_i N_j\left[\left(\frac{\partial}{\partial t}\overline{\rho u_i' u_j'} + \frac{\partial}{\partial x_k}\overline{\rho u_i' u_j' \tilde{u}_k}\right) + \left(\overline{\rho u_i' u_k'}\frac{\partial}{\partial x_k}\tilde{u}_j + \overline{\rho u_j' u_k'}\frac{\partial}{\partial x_k}\tilde{u}_i\right)\right]$$

It was shown (J. F. Debieve, 1978) that we can consider only the $\underline{/13}$ part of the tensions whose evolution is caused by production, specifically:

$$\frac{\partial}{\partial t}\overline{\rho u_i' u_j'} + \frac{\partial}{\partial x_k}\overline{\rho u_i' u_j' \tilde{u}_k} = -\overline{\rho u_i' u_k'}\frac{\partial}{\partial x_k}\tilde{u}_j - \overline{\rho u_j' u_k'}\frac{\partial}{\partial x_k}\tilde{u}_i$$

We obtain therefore:

$$\frac{D}{Dt}\overline{\rho(\underline{v}'\cdot\underline{N})^2}/\bar{\rho} = 0$$

$$\frac{\overline{\rho u_n'^2}}{\bar{\rho}} = \frac{C}{N^2}$$

C is a constant

$\overline{\rho u_n'^2}/\bar{\rho}$ corresponds to velocity fluctuations in the direction $\underline{N}$.

Thus we obtain a decrease of the level of the velocity fluctuations in the directions which dilate and conversely. As an example, let us examine the case of a single longitudinal gradient of stationary velocity (expansion or compression). In the transversal directions, the vector $\underline{N}$ remains constant, the transversal velocity fluctuations are unchanged. On the other hand, the longitudinal fluctuations will increase or decrease in the ratio $\frac{\overline{\rho u'^2}}{\bar{\rho}} = \frac{C^{te}}{U^2}$ We will therefore observe a decrease of $\frac{\overline{\rho u'^2}}{\bar{\rho}}$

in an expansion and an increase in compression.

These results reveal simply how the different Reynolds tensions may vary one with respect to the other under the effect of spatial variations of the average velocity field.

V. Discussion of the Validity of the Approximations Taken

V. 1. In paragraph II we showed the conditions in which it was possible to disregard the specific effects of turbulence. Two conditions were given:

$$\frac{q'}{\Delta U}\,\frac{L_d}{\Lambda_u} \ll 1$$

$$\frac{q'}{U}\,\frac{L_d}{\Lambda_u} \ll 1$$

The first inequality provides the assurance that we may disregard the non linear terms and those expressing the viscous effects, with the reservation that they retain the same order of magnitude in distortion, and upstream of the latter. If the second inequality applies it may be estimated that the dissipation rate will not be changed in the distorted area.

The order of magnitude of the diffusion term $\frac{\partial}{\partial x_i}\overline{\rho q'^2 u_i'}$ may be estimated:

$$\frac{\partial}{\partial x_i}\overline{\rho q'^2 u_i'} = \frac{\partial}{\partial x}\overline{\rho q'^2 u'} + \frac{\partial}{\partial y}\overline{\rho q'^2 v'}.$$

If the expansion considered were powerful enough to suppress all fluctuations, we would have:

$$\left|\frac{\partial}{\partial x}\overline{\rho q'^2 u'}\right| \sim \bar{\rho}\,\frac{q'^3}{L_d}$$

which probably furnishes an order of magnitude of the term considered.

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On the other hand, as regards the term $\frac{\partial}{\partial j} \overline{(q'^2 v')}$, it may be considered that it represents a diffusion of the kinetic energy of the turbulence $\overline{\rho q'^2}$ by the velocity fluctuation v'. This diffusion takes place over a distance comparable to an average dimension of the turbulent perturbations considered, for instance, the integral scale derived from spatial correlations or autocorrelation Λ_u . We have therefore

$$\frac{\partial}{\partial y} \overline{\rho q'^2 v'} \sim \bar{\rho} \frac{q'^3}{\Lambda_u}$$

In the studied case we find: $L_d \sim \delta$. If, moreover, Λ_u varies little in the expansion, we have $\Lambda_u \sim \delta \sim L_d$. The two diffusion terms are therefore really of the same order of magnitude as upstream of the expansion.

V. 2.   The application of the previously described inequalities implies that one should know a characteristic scale of the energy carrying turbulent perturbations Λ_u .

There are many measurements of these scales in subsonic flow, but little information is to be found in the literature on these scales in the supersonic boundary layer. Demetriades measured them in the case of a wake, with Mach number 3, and in a cooled boundary layer, Mach number 9. Preliminary measurements were therefore carried out at the I.M.S.T. (Institute of Statistical Mechanics of Turbulence) in the case of supersonic boundary layers.

The determination is carried out on the basis of the measurement with hot wire anemometer of the autocorrelation coefficient $R(\tau)$. We find by integration an integral time scale

$$\Lambda_i = \int_0^\infty R(\tau)\, d\tau$$

Λ_τ corresponds to the time of passage of a perturbation of magnitude Λ_u . If it is assumed that these perturbations undergo convection through the average flow with the local average value

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$\widetilde{u}_{\prime}$ we find

$$\Lambda_{u} = \widetilde{u} \, \Lambda_{\tau}$$

In the near wake flow, autocorrelation measurements of the signal given by the hot wire anemometer were carried out for an overheating coefficient 0.2. In this case, the anemometer is sensitive both to speed and temperature. The integral scales found are of the order 0.1δ (J. Gaviglio, J. P. Dussauge, J. F. Debieve, A. Favre, 1977).

More detailed measurements have been conducted in a boundary layer on plane plate for Mach number 1.8, by applying to the correlation measurements the technique of the "fluctuations diagram", which permits the separation of che respective effects of velocity and temperature. The preliminary results indicate that the integral velocity scale measured should be between $0.2\,\delta$ and $0.4\,\delta$. We recall that in subsonic boundary layer in equilibrium Λ_{u} is $0.45\,\delta$. With the results of the first measurements, we find that the parameter $\dfrac{q'}{U}\dfrac{L_d}{\Lambda_u}$ is between 0.1 and 0.3. It appears therefore that the dissipation rate hardly changes in the expansion area. If it is considered that the scale ΔU is of the order of the increase of the longitudinal velocity component in expansion $\dfrac{q'}{\Delta U}\dfrac{L_d}{\Lambda_{u}}$ is of the order of 1. Actually if we carry out a more detailed estimate of the term of production in expansion at a distance $(r - r_j)/\delta(o) \simeq 0.2$, we find that $-\overline{\dfrac{\rho' u_i' u_j'}{\bar\rho}}\dfrac{\partial \widetilde{u}_i}{\partial x_j} + \overline{\dfrac{\rho' u_i'}{\bar\rho^2}}\dfrac{\partial \bar p}{\partial x_i}$ is of the order $-0.3 \overline{u'^2}.u_\infty/\delta(o)$. The dissipation term assumed to be equal to the production term in the initial layer is of the order $0.1\,.\,\overline{u'^2}.u_\infty/\delta(o)$.

It seems therefore that in the expansion, the level reduction due to dissipation is not negligible. But it appears that the reduction observed in the levels does seem to have been caused mainly by the production of turbulence.

Another case may be examined: that of the compression undergone

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by the mixture layer near the reassembly (Fig. 1). This compression
is spread over a fairly long distance (2 to 3 times the thickness
of the turbulent layer) and we find in it considerable velocity
fluctuations whose intensity is of the order of 15%. In this <u>/17</u>
case the time of passage to compression is of the same order of
magnitude as the characteristic time of large turbulent structures.
Therefore the diffusion and dissipation effects must be taken into
consideration to describe even qualitatively the evolution of the
different turbulent flows.

VI. Conclusions

On the basis of the considerations on the characteristics
of the turbulence of clipped flows in the presence of high gradients
of average velocity and pressure, it was possible to reveal
conditions in which the evolution of the velocity fluctuations
does not depend totally on the specific properties of turbulence,
such as diffusion and dissipation, but especially on the effect of
the gradients of the average quantities. In an adapted experimental
case, the evolution of the statistical level of the velocity
fluctuations is interpreted as a function of the gradients of the
different average quantities. It was possible to relate this
analysis to a relaminarization coefficient. It was shown in
particular that a "relaminarization" is probably accompanied by
negative values of the term of turbulence production of kinetic
energy; a simple geometric representation of the effect of the
production of Reynolds tensions in the average velocity gradients
is given. Finally it was shown that in the case of the studied
experiment, although the effect of the production of turbulence
is predominant, the effect of energy dissipation by viscosity should
not be disregarded.

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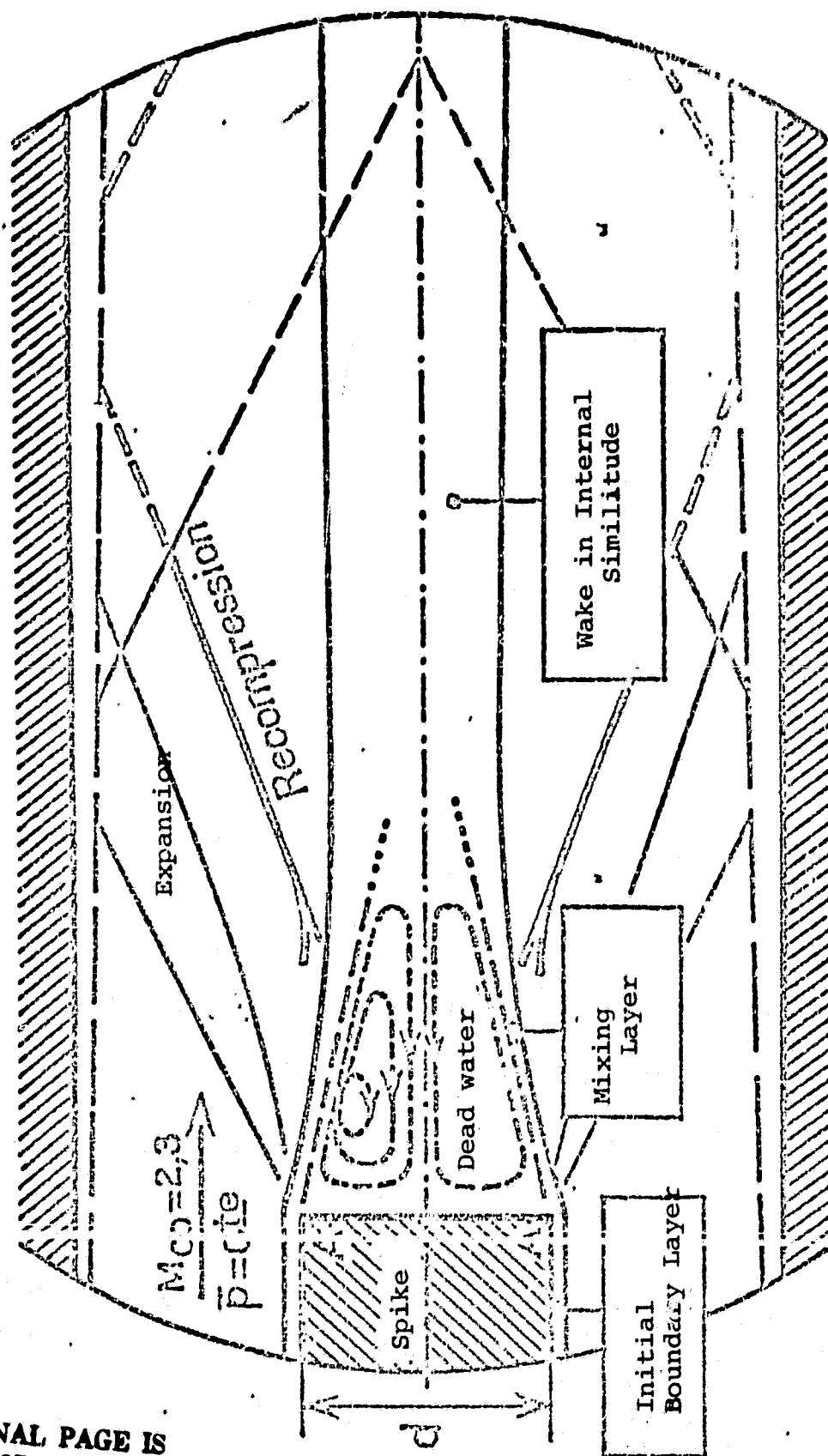29, avenue de la Division Leclerc
92320 Chatillon Sous Bagneux.

Figure 1. Scheme of the flow

Labels in figure:
$M_\infty = 2.3$
$\bar{p} = C^{te}$
Expansion
Recompression
Wake in Internal Similitude
Mixing Layer
Dead water
Spike
Initial Boundary Layer
d

		M_∞	$R_{\delta^{**}}$
o	JOHNSON ROSE	2.9	$4.7\,(10)^4$
△	KISTLER	1.72	$4\,(10)^4$
□	KISTLER	3.56	$3.3\,(10)^4$
---- KLEBANOFF		–	$7.75(10)^3$
—— ZORIC		–	$4.2\,(10)^4$
◇ I.M.S.T.			$(10)^3$

Hot Wire

LASER o JOHNSON ROSE 2.9 $4.7\,(10)^4$

Figure 2.

$$\left(\frac{\overline{\rho u'^2}}{\tau_0}\right)^{\!1/2}$$

$$\frac{r - r_j}{\delta}$$
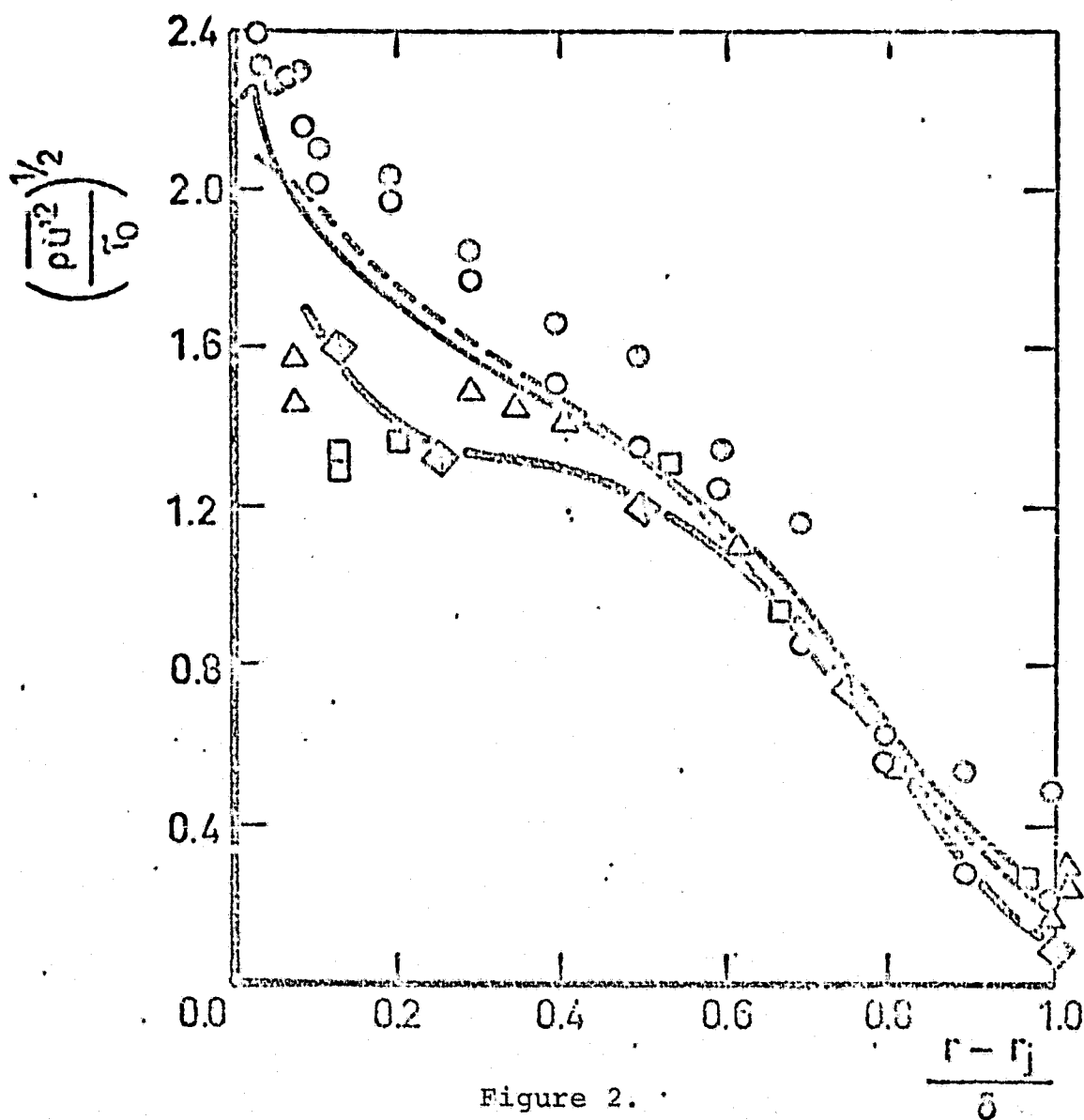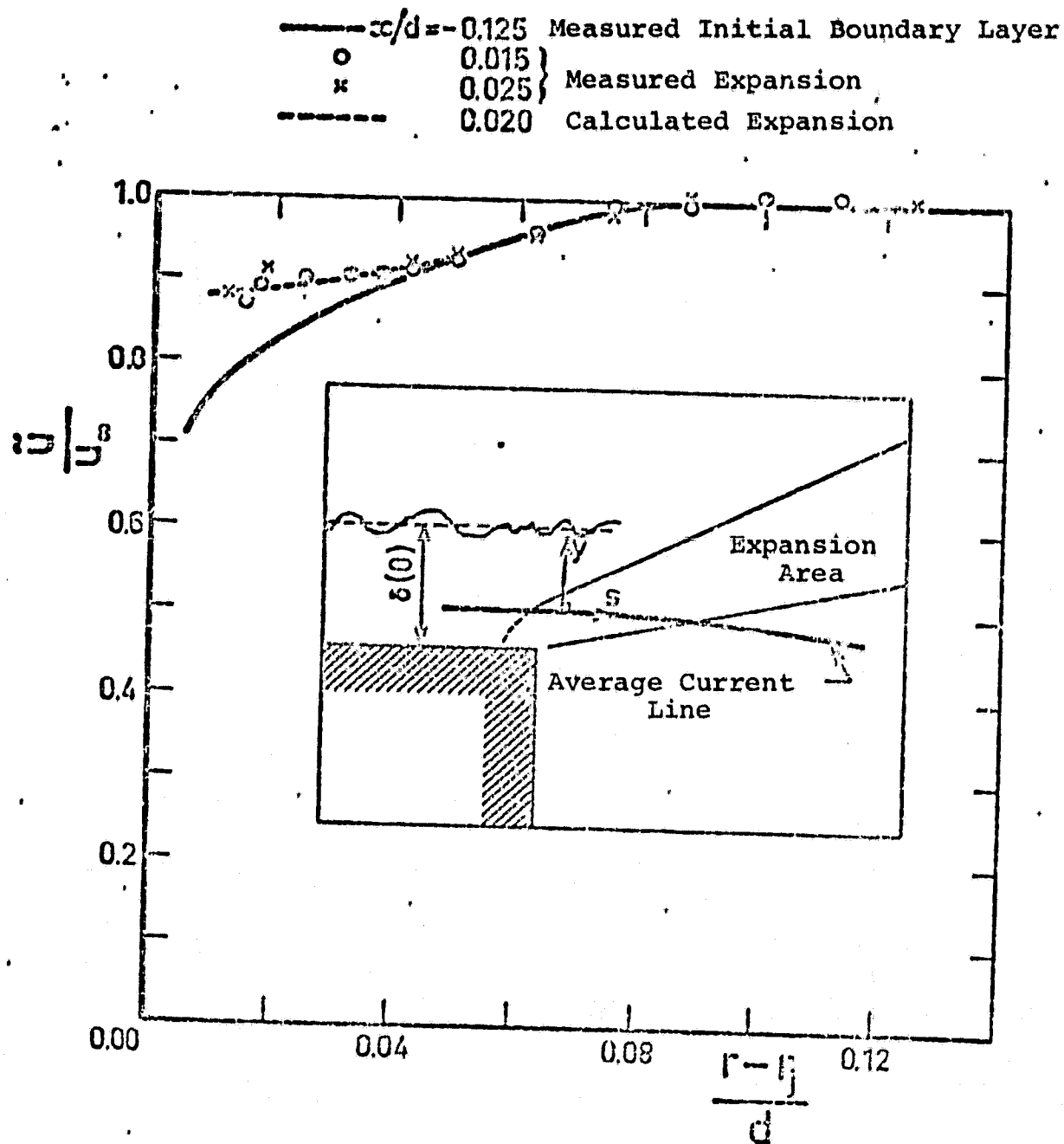
Figure 3. Average Velocity Profiles
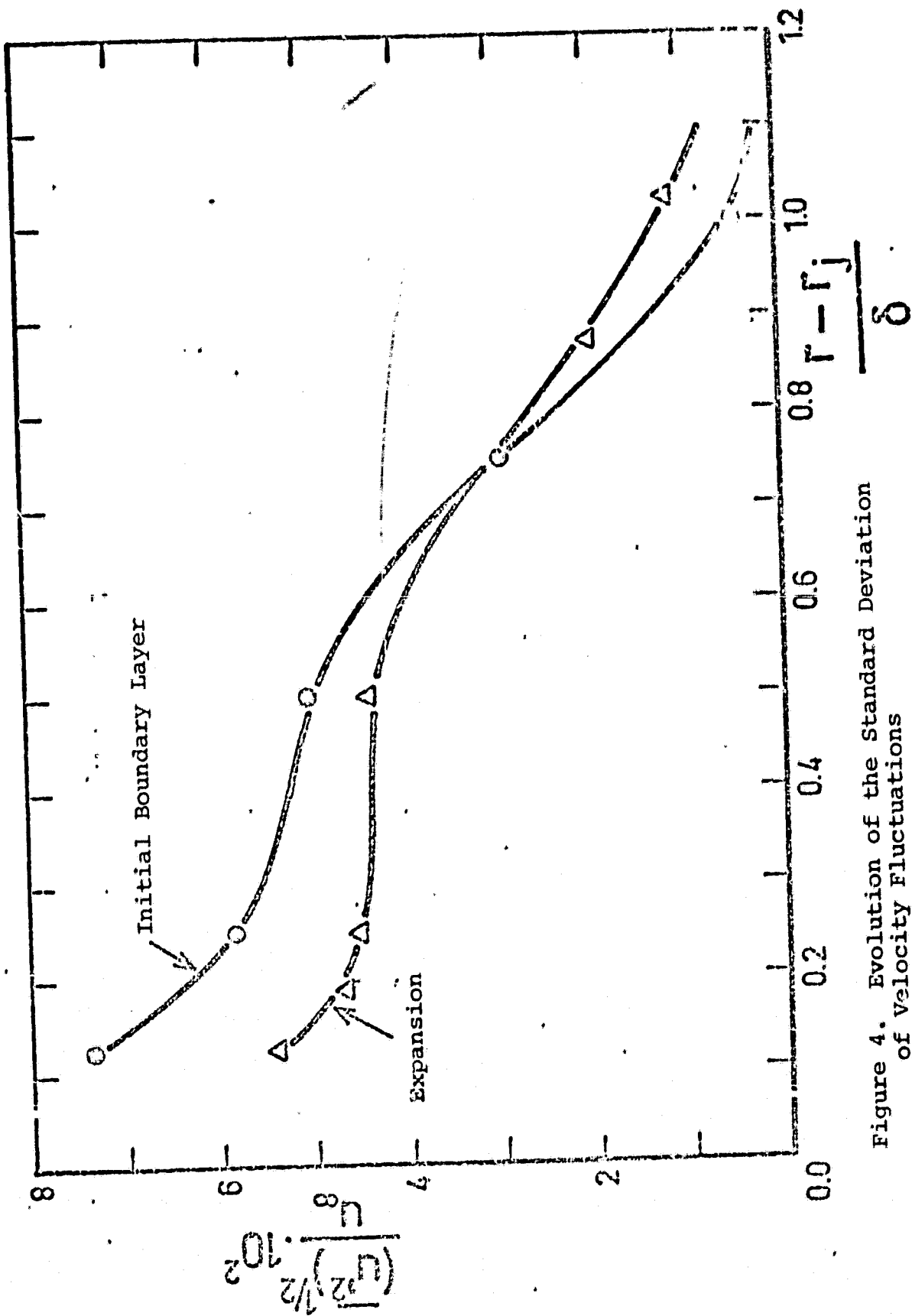
Figure 4. Evolution of the Standard Deviation of Velocity Fluctuations
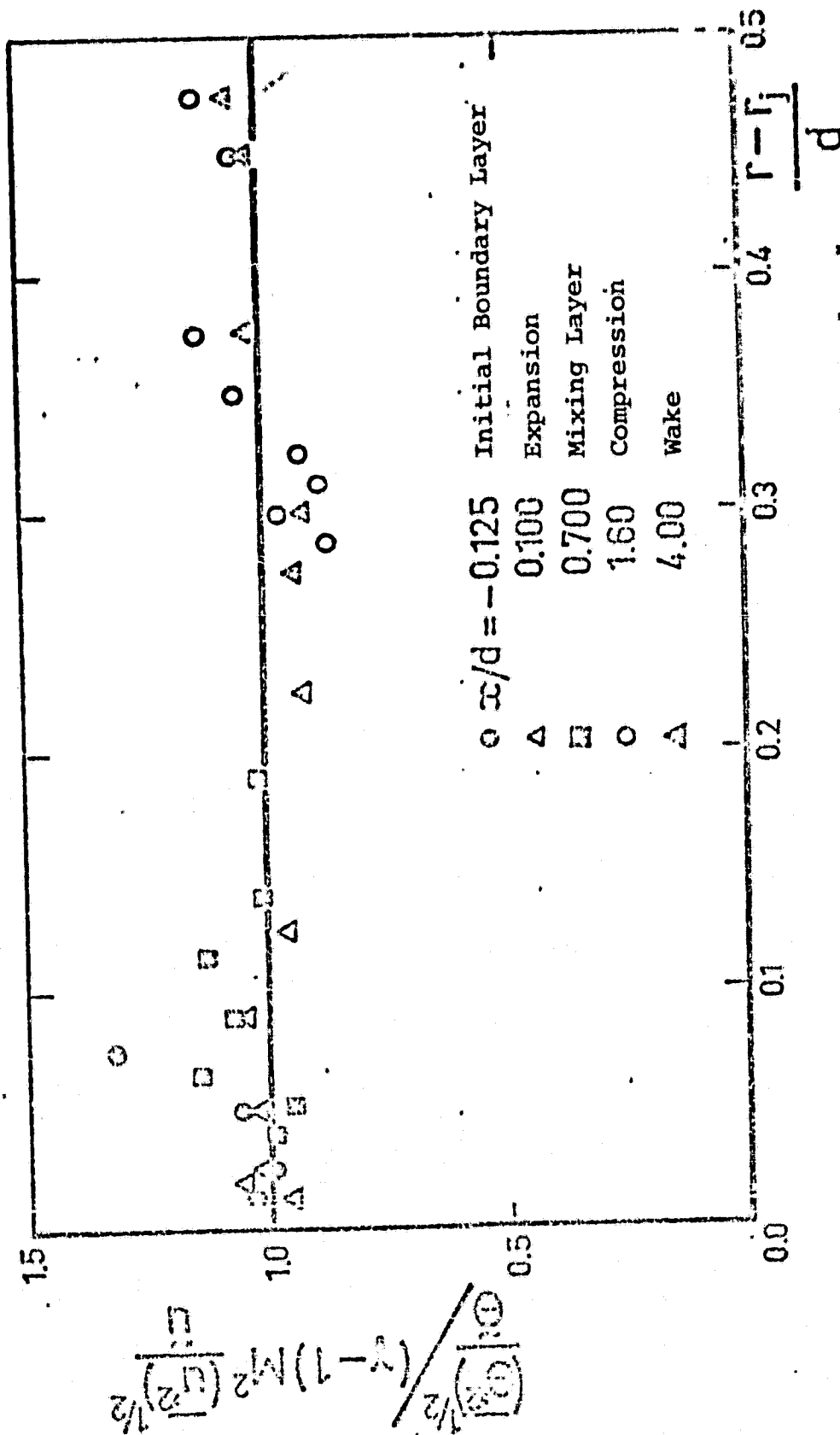
Figure 5. Verification of "Strong Reynolds Analogy"

$x/d = -0.125$ Initial Boundary Layer

0.100 Expansion
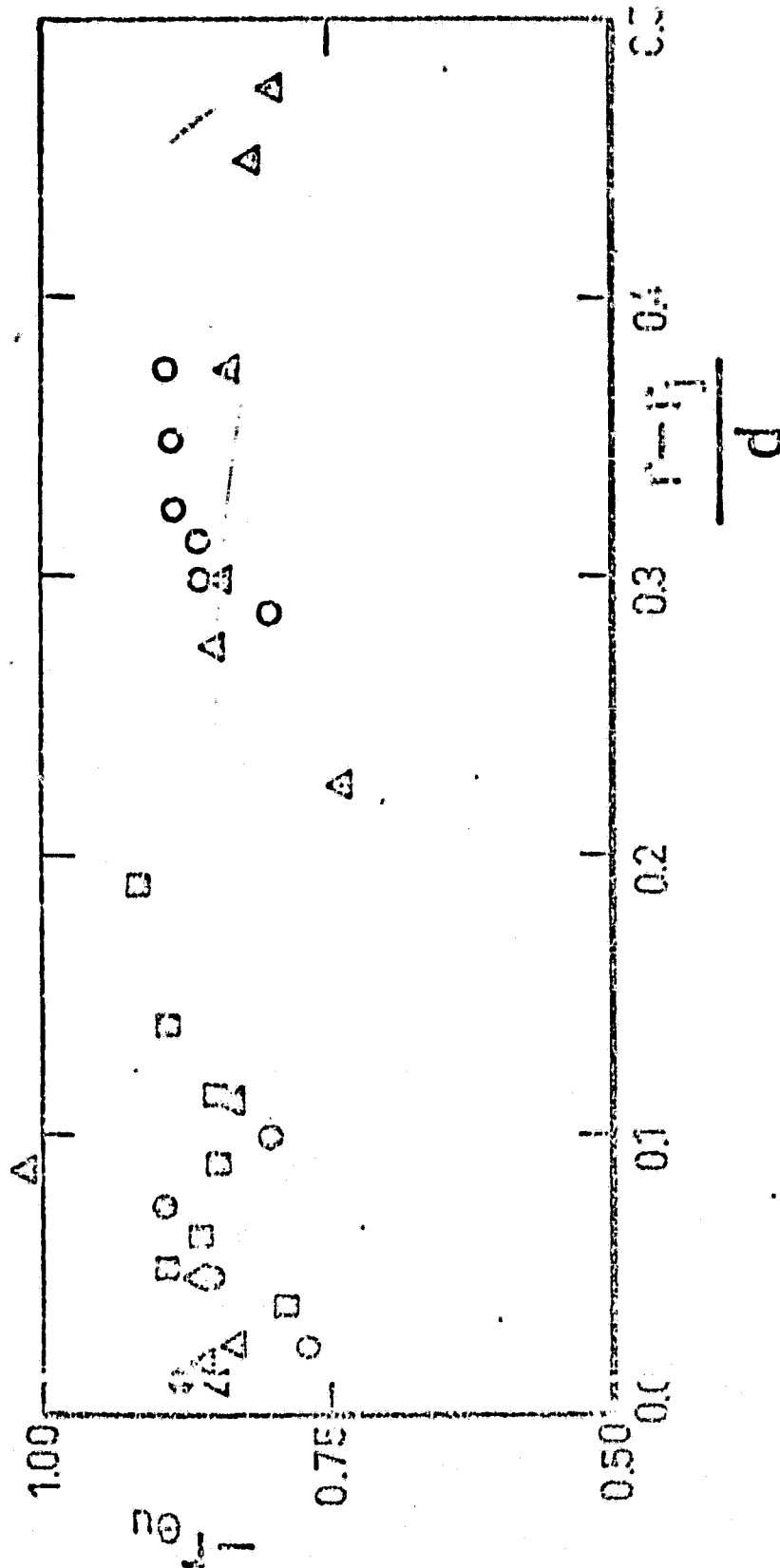
0.700 Mixing Layer

1.60 Compression

4.00 Wake

$-R_{u\theta}$

1.00

0.75

0.50

0.0   0.1   0.2   0.3   0.4   0.5

$$\frac{i - i_1}{d}$$

Figure 6. Correlation Coefficient Between Velocity and Temperature

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